Journal of Engineering Physics and Thermophysics, Vol. 77, No. 6, 2004

INTERRELATION BETWEEN MOVING FORCES IN SOME MODELS AND FORMS OF FLOW ORGANIZATION

V. N. Pavlechko

UDC 66.048.375

The equality of the number of transfer units N in some models and variants of mass transfer and also for different forms of organization of vapor and liquid flows has been proved despite the difference in the expressions for N and quantities of mass-transfer efficiencies. Theoretical computations are confirmed by practical calculations. The constancy of the number of transfer units for all plates at the same operating parameters and closeness of logarithmic-mean and arithmetic-mean values of N are shown.

It is known from [1] that the number of transfer units in the vapor phase in concurrent motion for variants using the conditions of interrelation between the ideal and actual trays of the Murphree [2] and Hausen [3] models are

$$N_{\text{con,v1}} = \frac{\ln \dfrac{1 + \dfrac{mV}{L} E_{\text{con1}}}{1 - E_{\text{con1}}}}{1 + \dfrac{mV}{L}} , \quad (1)$$

$$N_{\text{con,v2}} = \frac{\ln \dfrac{1 + \dfrac{L}{mV} E_{\text{con2}}}{1 - E_{\text{con2}}}}{1 + \dfrac{mV}{L}} , \quad (2)$$

$$N_{\text{con,v3}} = \frac{\ln \dfrac{1}{1 - E_{\text{con3}}}}{1 + \dfrac{mV}{L}} . \quad (3)$$

A similar dependence was also derived in the complex model [4]

$$N_{\text{con,v}} = \frac{\ln \dfrac{1 + \dfrac{E_{\text{con}}}{m}}{1 - E_{\text{con}}}}{1 + \dfrac{mV}{L}} . \quad (4)$$

In the case of a linear equilibrium ratio within the tray, the efficiencies according to Murphree and Hausen are related by the equation [5–7]

Belorussian State Technological University, 13a Sverdlov Str., Minsk, 220050, Belarus; email: Paulechka@tut.by. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 77, No. 6, pp. 90–96, November–December, 2004. Original article submitted September 25, 2003.

$$\frac{1}{E_{\mathrm{M,v}}} + \frac{mV}{L} = \frac{\frac{mV}{L}}{E_{\mathrm{M,liq}}} + 1 = \frac{1 + \frac{mV}{L}}{E_{\mathrm{H}}} . \tag{5}$$

This relation in a somewhat modified form, when the mass-transfer models are considered as variants in concurrent motion [8] — the first of these variants reflects the conditions of interrelation between the ideal and actual plates typical of the Murphree model in analysis of the efficiency in the vapor phase; the second variant reflects the conditions of interrelation between these plates typical of the Murphree model in analysis of the efficiency in liquid; the third variant reflects the conditions of interrelation between these plates trays typical of the Hausen model with account for the complex model [9] — is presented as

$$\frac{\frac{L}{mV}}{E_{\mathrm{con1}}} = \frac{L}{mV} + \frac{1}{E_{\mathrm{con2}}} - 1 = \frac{\frac{L}{mV} + 1}{E_{\mathrm{con3}}} - 1 = \frac{\frac{L}{V} + m}{(m+1)\, E_{\mathrm{con}}} + \frac{\frac{L}{mV} - m}{m+1} . \tag{6}$$

We express the efficiencies E_{con2}, E_{con3}, and E_{con} from (6) in terms of E_{con1}:

$$E_{\mathrm{con2}} = \frac{E_{\mathrm{con1}}}{\frac{L}{mV} - E_{\mathrm{con1}}\left(\frac{L}{mV} - 1\right)} , \tag{7}$$

$$E_{\mathrm{con3}} = \frac{\frac{L}{mV} - 1}{\frac{L}{mV} + E_{\mathrm{con1}}} E_{\mathrm{con1}} , \tag{8}$$

$$E_{\mathrm{con}} = \frac{\frac{L}{V} + m}{\frac{L}{mV} - E_{\mathrm{con1}}\left(\frac{L}{mV} - m\right)} E_{\mathrm{con1}} . \tag{9}$$

In substituting relations (7)–(9) into formulas (2)–(4), we obtain the corresponding expressions similar to Eq. (1), which indicates the equality of the number of transfer units in different models (variants) of mass transfer:

$$N_{\mathrm{con,v1}} = N_{\mathrm{con,v2}} = N_{\mathrm{con,v3}} = N_{\mathrm{con,v}} . \tag{10}$$

Relations similar to (6) are also obtained for countercurrent motion and cross motion of vapor and liquid from the analysis of [8, 10] and [8, 11], respectively:

$$\frac{\frac{L}{mV} - 1}{E_{\mathrm{g1}}} = \frac{\frac{L}{mV}}{E_{\mathrm{g3}}} - 1 = \frac{L}{mV} - \frac{1}{E_{\mathrm{g4}}} = \frac{\frac{L}{V} - 1}{(m+1)\, E_{\mathrm{g}}} + \frac{\frac{L}{mV} - m}{m+1} , \tag{11}$$

$$\frac{\frac{L}{mV} - \frac{1}{2}}{E_{\mathrm{k1}}} = \frac{L}{mV} - 1 + \frac{1}{2E_{\mathrm{k2}}} = \frac{\frac{L}{mV} + \frac{1}{2}}{E_{\mathrm{k3}}} - 1 = \frac{L}{mV} - \frac{1}{2E_{\mathrm{k4}}} = \frac{\frac{L}{V} + \frac{m-1}{2}}{(m+1)\, E_{\mathrm{k}}} + \frac{\frac{L}{mV} + m}{m+1} . \tag{12}$$

Combined solution of the expressions for the number of transfer units for countercurrent motion and cross motion from Table 3 [1] and correspondingly from (11) and (12) leads to the equations

$$N_{\mathrm{g,v1}} = N_{\mathrm{g,v3}} = N_{\mathrm{g,v4}} = N_{\mathrm{g,v}} , \tag{13}$$

$$N_{k,v1} = N_{k,v2} = N_{k,v3} = N_{k,v4} = N_{k,v} . \tag{14}$$

Since the numbers of transfer units in the vapor and liquid phases are related by the expression

$$N_{liq} = \frac{mV}{L} N_v , \tag{15}$$

from (10), (13), and (14) we also obtain the equalities of the numbers of transfer units in the liquid phase.

Thus, the number of transfer units does not depend on the model of mass transfer and is determined only by the parameters of the medium being separated and operating and structural factors.

Of interest is also the interrelation between the number of transfer units for different forms of organization of vapor and liquid flows. Comparison of values of N from [1] for the variants using the conditions of interrelation between the ideal and actual trays typical of the Murphree and Hausen models and moving forces from [4] for the complex model has certain mathematical difficulties due to the use of a logarithmic function in them. Therefore, it is expedient to refer to arithmetic-mean moving forces. This reference is admissible if the ratio of mean moving forces does not fall outside the limits of 0.5–2. For concurrent motion, which is characterized by the greatest difference between the initial and final moving forces, this ratio is equal to the number of the logarithm in (4) and must not exceed

$$\frac{1 + \dfrac{E_{con}}{m}}{1 - E_{con}} \le 2 , \tag{16}$$

whence

$$E_{con} \le \frac{m}{2m + 1} . \tag{17}$$

Thus, substitution of logarithmic-mean moving forces by arithmetic-mean ones is possible provided that condition (17) is met. In particular, for mixtures approaching ideal ones ($m \to 1$), the efficiency of mass transfer must not exceed 0.333, for the furfurol–water mixture — 0.47, and for the ethanol–water mixture — 0.48 and at an infinitely large coefficient of phase equilibrium — 0.5. In countercurrent motion, the use of arithmetic-mean values is more expedient than the use of logarithmic-mean ones, since calculation of the latter is difficult and becomes impossible when the working and equilibrium lines are parallel. Cross motion occupies, in this sense, an intermediate position between concurrent and countercurrent motions.

In the complex model, arithmetic-mean numbers of transfer units in liquid for concurrent motion, countercurrent motion, and cross motion, respectively, have the form

$$N_{con,liq} = \frac{(m+1)\, E_{con}}{\dfrac{L}{V} + m - \dfrac{E_{con}}{2}\left(\dfrac{L}{mV} + 1\right)(m - 1)} , \tag{18}$$

$$N_{g,liq} = \frac{(m+1)\, E_g}{\dfrac{L}{V} - 1 - \dfrac{E_g}{2}\left(\dfrac{L}{mV} + 1\right)(m - 1)} , \tag{19}$$

$$N_{k,liq} = \frac{(m+1)\, E_k}{\dfrac{L}{V} + \dfrac{m - 1}{2} - \dfrac{E_k}{2}\left(\dfrac{L}{mV} + 1\right)(m - 1)} . \tag{20}$$

Equating the right-hand sides of formulas (1), (3), and (5), from [4] we obtain the ratio of the efficiencies for different forms of organization of flows in the complex model:

$$\frac{\frac{L}{mV}+m}{E_{\text{con}}}=\frac{\frac{L}{mV}-1}{E_{\text{g}}}=\frac{\frac{L}{mV}+\frac{m-1}{2}}{E_{\text{k}}}\,. \tag{21}$$

We express the efficiencies E_{g} and E_{k} in terms of E_{con} from (21):

$$E_{\text{g}}=\frac{\frac{L}{V}-1}{\frac{L}{V}+m}E_{\text{con}}\,, \tag{22}$$

$$E_{\text{k}}=\frac{\frac{L}{V}+\frac{m-1}{2}}{\frac{L}{V}+m}E_{\text{con}}\,, \tag{23}$$

and, having substituted them in (19) and (20), respectively, we obtain

$$N_{\text{g,liq}}=\frac{(m+1)\,E_{\text{con}}}{\frac{L}{V}+m-\frac{E_{\text{con}}}{2}\left(\frac{L}{mV}+1\right)(m-1)}\,, \tag{24}$$

$$N_{\text{k,liq}}=\frac{(m+1)\,E_{\text{con}}}{\frac{L}{V}+m-\frac{E_{\text{con}}}{2}\left(\frac{L}{mV}+1\right)(m-1)}\,. \tag{25}$$

The coincidence of the right-hand sides of (18), (24), and (25) proves the equality of their left-hand sides, which allows one to write

$$N_{\text{con,liq}}=N_{\text{g,liq}}=N_{\text{k,liq}}\,. \tag{26}$$

The same results were obtained for the Murphree and Hausen models and for the variants that use the conditions of interrelation between the ideal and actual plates, which are typical of these models, in countercurrent and cross motion, which leads to equalities in both the liquid and vapor phases

$$N_{\text{con1}}=N_{\text{g1}}=N_{\text{k1}}\,,\quad N_{\text{con2}}=N_{\text{k2}}\,,\quad N_{\text{con3}}=N_{\text{g3}}=N_{\text{k3}}\,,\quad N_{\text{g4}}=N_{\text{k4}}\,. \tag{27}$$

The absence of N for the fourth variant in concurrent motion and for the second variant in countercurrent motion is caused by the indeterminance of efficiency in these variants. The equality of the number of transfer units in (26) and (27) indicates that these quantities for the considered models of mass transfer are also independent of the relation between the directions of vapor and liquid flows.

The validity of the computations was checked by the commercial data obtained in separation of the furfurol–water mixture on the main furfurol column at the Bendery Biochemical Plant. The rectification column had 25 cup plates in its output part and operated in the regime of cross motion of the vapor and liquid phases. The column is calculated by the dependences for both cross motion and concurrent motion, assuming complete mixing of the liquid on the tray.

In the calculations by the initial data, the coefficient of phase equilibrium and compositions of vapor and liquid above the first plate from below were determined at an arbitrary value of efficiency. Then the values of m were

TABLE 1. Concluding Results of Calculation of the Number of Transfer Units

Plate No.	$x \cdot 10^3$, mole %	$y \cdot 10^3$, mole %	m	$N_{liq,log}$	$N_{liq,a}$
Cube	0.0488	0.0000	7.984	—	—
1	0.0572	0.0339	7.981	0.1665	0.1657
2	0.0663	0.0706	7.978	0.1665	0.1657
3	0.0763	0.1105	7.975	0.1665	0.1656
4	0.0870	0.1538	7.971	0.1665	0.1656
5	0.0987	0.2008	7.967	0.1665	0.1656
6	0.1114	0.2519	7.963	0.1665	0.1656
7	0.1252	0.3072	7.958	0.1664	0.1656
8	0.1402	0.3672	7.953	0.1664	0.1656
9	0.1564	0.4325	7.948	0.1664	0.1655
10	0.1740	0.5032	7.942	0.1664	0.1655
11	0.1931	0.5799	7.936	0.1663	0.1655
12	0.2138	0.6631	7.929	0.1663	0.1654
13	0.2362	0.7533	7.922	0.1663	0.1654
14	0.2606	0.8510	7.914	0.1662	0.1654
15	0.2869	0.9570	7.905	0.1662	0.1653
16	0.3155	1.0718	7.896	0.1662	0.1653
17	0.3465	1.1962	7.886	0.1661	0.1652
18	0.3800	1.3310	7.875	0.1661	0.1652
19	0.4163	1.4769	7.863	0.1660	0.1651
20	0.4556	1.6348	7.850	0.1659	0.1651
21	0.4982	1.8057	7.837	0.1659	0.1650
22	0.5442	1.9906	7.822	0.1658	0.1649
23	0.5939	2.1905	7.806	0.1657	0.1649
24	0.6477	2.4066	7.789	0.1657	0.1648
25	0.7058	2.6401	7.770	0.1656	0.1647

TABLE 2. Efficiencies and Logarithmic-Mean Numbers of Transfer Units in Concurrent Motion in Liquid

Test No.	L/V	x_{feed}, wt.%	x_{cube}, wt.%	E_{con} $N_{con,log}$	E_{con1} $N_{con1,log}$	E_{con2} $N_{con2,log}$	E_{con3} $N_{con3,log}$
1	4.018	0.80	0.026	0.2675 0.2281	0.1223 0.2293	0.2140 0.2293	0.2916 0.2293
2	4.149	0.74	0.064	0.1904 0.1537	0.0842 0.1546	0.1477 0.1547	0.2096 0.1546
3	4.302	0.42	0.026	0.2250 0.1832	0.1033 0.1838	0.1748 0.1839	0.2465 0.1839
4	4.345	0.61	0.028	0.2559 0.2111	0.1211 0.2120	0.1998 0.2121	0.2793 0.2121
5	4.470	0.37	0.036	0.1901 0.1499	0.0872 0.1504	0.1450 0.1509	0.2091 0.1503
6	4.470	0.58	0.040	0.2245 0.1800	0.1052 0.1809	0.1722 0.1809	0.2461 0.1810
7	4.662	0.50	0.059	0.1796 0.1384	0.0841 0.1391	0.1341 0.1391	0.1979 0.1391
8	4.782	0.64	0.060	0.2094 0.1623	0.1017 0.1633	0.1564 0.1632	0.2299 0.1632
9	4.974	0.45	0.077	0.1513 0.1121	0.0722 0.1126	0.1096 0.1127	0.1673 0.1126
10	5.175	0.71	0.087	0.1981 0.1478	0.1000 0.1488	0.1435 0.1489	0.2179 0.1488

TABLE 3. Efficiencies and Logarithmic-Mean Numbers of Transfer Units in Cross Motion in Liquid

Test No.	L/V	x_{feed}, wt.%	x_{cube}, wt.%	E_k $N_{k,log}$	E_{k1} $N_{k1,log}$	E_{k2} $N_{k2,log}$	E_{k3} $N_{k3,log}$
1	4.018	0.80	0.026	0.1715 0.2300	0.0053 0.2292	0.1053 0.2222	0.1940 0.2222
2	4.149	0.74	0.064	0.1233 0.1559	0.0070 0.1557	0.0724 0.1498	0.1400 0.1498
3	4.302	0.42	0.026	0.1450 0.1845	0.0098 0.1846	0.0865 0.1801	0.1660 0.1802
4	4.345	0.61	0.028	0.1663 0.2128	0.0136 0.2124	0.0987 0.2066	0.1884 0.2064
5	4.470	0.37	0.036	0.1234 0.1508	0.0113 0.1510	0.0715 0.1476	0.1417 0.1476
6	4.470	0.58	0.04	0.1470 0.1821	0.0149 0.1825	0.0849 0.1763	0.1667 0.1762
7	4.662	0.50	0.059	0.1185 0.1401	0.0149 0.1401	0.0661 0.1358	0.1350 0.1358
8	4.782	0.64	0.060	0.1400 0.1651	0.0211 0.1654	0.0769 0.1585	0.1575 0.1584
9	4.974	0.45	0.077	0.1011 0.1134	0.0167 0.1136	0.0539 0.1100	0.1154 0.1100
10	5.175	0.71	0.087	0.1358 0.1514	0.0221 0.1515	0.0701 0.1435	0.1511 0.1435

verified and the concentrations of vapor and liquid above the second plate from below were determined at the same value of efficiency [12]. Calculations were made successively for the third and remaining plates, including the feeding plate, and upon reaching the last plate the calculated and specified concentrations of furfurol in the feeding liquid were compared. If the calculated and experimental values differed, the efficiency was corrected and calculations continued in the same order until an acceptable coincidence of concentrations on the feeding plates was reached.

Upon determination of the efficiency, the numbers of transfer units were calculated by expressions (1)–(4) for concurrent motion and by the dependences given in [4] for cross motion. As an example, Table 1 gives results of the calculation of the complex model in concurrent motion for the following parameters: L/V = 4.018, E = 0.2018, x_{feed} = 0.375 wt.%, x_{cube} = 0.026 wt.%, and y_{cube} = 0 due to heating of the column by live vapor.

As is seen from Table 1, the numbers of transfer units remain virtually constant for all plates of the output part of the column, slightly decreasing with a decrease in m. Moreover, logarithmic-mean values of N practically do not differ from the corresponding arithmetic-mean values. Here, the mentioned uniformity takes place for other similar parameters. This is due to the fact that efficiencies of mass transfer are more than twice as small as the a limiting value regulated by formula (17).

The results of calculation of the main furfurol column in concurrent and cross motion are given in Tables 2 and 3, respectively. These tables present mean values of N for all 25 calculated plates. Slight deviations of the quantities for the same parameters are due, probably, to errors of rounding in calculation of the efficiencies. The tabulated data demonstrate the closeness of the number of transfer units for the Murphree (1st and 2nd variants) and Hausen (3rd variant) models in concurrent motion (Table 2) and the variants that involve the conditions of interrelation between the ideal and actual plates of the indicated models united by the complex model, including them as limiting cases, in cross motion (Table 3).

Comparison of the data from Tables 2 and 4 shows the equality of logarithmic-mean and arithmetic-mean numbers of transfer units even in concurrent motion, as is observed in Table 1.

Similar results are also obtained in using commercial data on separation of a furfurol–water mixture at 23 enterprises dealing with hydrolysis, separation of an ethanol–water mixture at the Bobruisk hydrolysis plant, and concur-

TABLE 4. Efficiencies and Arithmetic-Mean Numbers of Transfer Units in Concurrent Motion in Liquid

Test No.	L/V	x_{feed}, wt.%	x_{cube}, wt.%	E_{con} / $N_{con,a}$	E_{con1} / $N_{con1,a}$	E_{con2} / $N_{con2,a}$	E_{con3} / $N_{con3,a}$
1	4.018	0.80	0.026	0.2675 0.2258	0.1223 0.2258	0.2140 0.2258	0.2916 0.2258
2	4.149	0.74	0.064	0.1904 0.1530	0.0842 0.1530	0.1477 0.1530	0.2096 0.1530
3	4.302	0.42	0.026	0.2250 0.1820	0.1033 0.1820	0.1748 0.1821	0.2465 0.1821
4	4.345	0.61	0.028	0.2559 0.2092	0.1211 0.2091	0.1998 0.2092	0.2793 0.2093
5	4.470	0.37	0.036	0.1901 0.1492	0.0872 0.1492	0.1450 0.1497	0.2091 0.1492
6	4.470	0.58	0.040	0.2245 0.1789	0.1052 0.1780	0.1722 0.1788	0.2461 0.1789
7	4.662	0.50	0.059	0.1796 0.1379	0.0841 0.1379	0.1341 0.1379	0.1979 0.1379
8	4.782	0.64	0.060	0.2094 0.1614	0.1017 0.1614	0.1564 0.1613	0.2299 0.1613
9	4.974	0.45	0.077	0.1513 0.1118	0.0722 0.1118	0.1096 0.1119	0.1673 0.1118
10	5.175	0.71	0.087	0.1981 0.1471	0.1000 0.1470	0.1435 0.1470	0.2179 0.1470

rent separation of a water–acetic acid mixture under experimental conditions [12]. However, due to the uniformity of the results obtained these data are not presented.

Thus, the numbers of transfer units, despite the difference in their expressions and values of mass-transfer efficiencies, have the same values for the same operating parameters and are independent of either the considered models or the flow structure of the vapor and liquid phases. By virtue of this, it is expedient to consider N as an example of a mass-transfer parameter that is more universal compared with efficiency.

NOTATION

E, efficiency of mass transfer; L, molar liquid flow; m, coefficient of phase equilibrium; N, number of transfer units; V, molar vapor flow; x and y, concentration of an easily volatile component in the liquid and vapor phases, respectively. Indices: a, arithmetic-mean value; g, countercurrent motion; H, Hausen model; cube, in the cube of the column; k, cross motion; log, logarithmic-mean value; liq, liquid phase; M, Murphree model; n, number of the considered plate; $n-1$, number of the previous plate toward vapor motion; con, concurrent motion; feed, on the feeding plate; v, vapor phase; 1–4, variants of mass transfer.

REFERENCES

1. V. N. Pavlechko, I. M. Plekhov, and V. N. Gulyaev, Interrelation between the kinetic indices in the process of rectification and moving forces, *Inzh.-Fiz. Zh.*, **74**, No. 5, 171–176 (2001).
2. E. V. Murphree, Rectifying column calculation with particular reference to *n*-component mixtures, *Ind. Eng. Chem.*, **17**, No. 7, 747–750 (1925).
3. H. Hausen, Zur Definition des Austauschgrades von Rektifizierböden bei Zwei- und Dreistoff-Gemischen, *Chem. Ing. Tech.*, **25**, No. 10, 595–597 (1953).

4. V. N. Pavlechko, Motive force in a complex model of mass transfer, *Inzh.-Fiz. Zh.*, **77**, No. 1, 96–100 (2004).
5. A. G. Medina, N. Ashton, and C. McDermott, Hausen and Murphree efficiencies in binary and multicomponent distillation, *Chem. Eng. Sci.*, **34**, No. 9, 1105–1112 (1979).
6. N. I. Savel'ev and N. A. Nikolaev, Methods for expressing the efficiency of contact devices of mass exchangers and interrelation between them, *Izv. Vyssh. Uchebn. Zaved., Khim. Khim. Tekhnol.*, **28**, No. 9, 95–98 (1985).
7. N. I. Savel'ev and N. A. Nikolaev, Mathematical description and analysis of laws governing mass exchange in contact devices with straight-twisted motion of phases, *Teor. Osnovy Khim. Tekhnol.*, **13**, No. 4, 435–444 (1989).
8. V. N. Pavlechko, Analysis of interrelations between efficiencies in basic models of mass transfer, in: *Proc. Int. Sci.-Tech. Conf.* "*Resource- and Energy-Saving Technologies in the Chemical Industry and Manufacture of Building Materials*" [in Russian], Minsk (2000), pp. 74–76.
9. V. N. Pavlechko, Complex model of the efficiency of rectification plates. 1. Concurrent motion of the phases, *Inzh.-Fiz. Zh.*, **74**, No. 1, 50–56 (2001).
10. V. N. Pavlechko, Complex model of the efficiency of rectification plates. 2. Countercurrent motion of the phases, *Inzh.-Fiz. Zh.*, **74**, No. 1, 57–61 (2001).
11. V. N. Pavlechko, Complex model of the efficiency of rectification plates. 4. Cross motion of phases, *Inzh.-Fiz. Zh.*, **74**, No. 2, 43–47 (2001).
12. V. N. Pavlechko and É. I. Levdanskii, Complex model of the efficiency of rectification plates. 8. Comparison with other models with respect to experimental data, *Inzh.-Fiz. Zh.*, **75**, No. 3, 17–21 (2002).